

Don't forget to measure Δs

Stephen F. Pate

Physics Department, New Mexico State University, Las Cruces NM 88003, USA

Received: 15 October 2004 / Published Online: 8 February 2005
 © Società Italiana di Fisica / Springer-Verlag 2005

Abstract. This talk explores our lack of knowledge of the strange quark contribution to the nucleon spin, Δs . Data on Δs from inclusive and semi-inclusive polarized deep-inelastic scattering will be reviewed, followed by a discussion of how the ongoing program of parity-violating elastic electron-nucleon scattering experiments, that seek out the strange electromagnetic form factors of the nucleon, need to have an estimate for the strange axial form factor to carry out that program, and how the value of Δs extracted from the DIS experiments has filled that role. It is shown that elastic νp , $\bar{\nu} p$, and parity-violating ep data can be combined to extract the strange electric, magnetic *and* axial form factors simultaneously. A proposed experiment that could address this important issue is briefly previewed.

PACS. 14.20.Dh Protons and neutrons – 13.40.Gp Electromagnetic form factors of elementary particles

1 Introduction

In the current experimental program of nucleon structure studies, we find two broad areas of experimentation. On the one hand, elastic scattering of electrons from nucleons is used to measure the electromagnetic and axial form factors of the nucleon, over a range of momentum transfer of $0.1 < Q^2 < 10$ (GeV/c)². These experiments have taken place at a variety of laboratories over the years, with the current program focused at MIT-Bates, JLab, and Mainz. One of the highlights of the current program is the emphasis on nailing down the strange quark contributions to the electromagnetic form factors, through the exploitation of the interference between photon and Z -boson exchange processes. On the other hand, deep-inelastic scattering of muons and electrons from nucleon and nuclear targets, historically responsible for the discovery of the partonic structure of matter, continues to play a role in the exploration of the distribution of quarks and gluons in nucleons. One of the highlights here is the focus, over the last 15 years, on the spin structure of the nucleon. The deep-inelastic exploration of nucleon spin takes place now at both leptonic and hadronic facilities, the spin program at RHIC being the most notable example of an hadronic facility taking on this physics topic.

QCD provides a simple framework in which these two experimental programs are joined together. The asymmetries observed in the polarized deep-inelastic scattering experiments arise from the antisymmetric part of the virtual Compton amplitude, which contains at its heart the nucleon axial current, $\bar{q}\gamma_\mu\gamma_5q$. In the quark-parton model, inclusive scattering of leptons from nucleon targets measures the nucleon structure function F_1 ,

$$F_1(x) = \frac{1}{2} \sum_q e_q^2 q(x)$$

where e_q and $q(x)$ are respectively the charge and parton distribution function for quarks of flavor q . Inclusive scattering of *polarized* leptons from *polarized* nucleon targets measures the *spin-dependent* nucleon structure function g_1 ,

$$g_1(x) = \frac{1}{2} \sum_q e_q^2 \Delta q(x)$$

where now $\Delta q(x)$ is a polarized parton distribution function. In QCD, these distribution functions take on a scale dependence: $\Delta q(x, Q^2)$. At the same time, the axial form factors $G_A^q(Q^2)$ measured in elastic scattering are themselves matrix elements of the axial current,

$${}_N \langle p' | \bar{q} \gamma_\mu \gamma_5 q | p \rangle_N = \bar{u}(p') \gamma_\mu \gamma_5 G_A^q(Q^2) u(p)$$

where the matrix element has been taken between two nucleon states of momenta p and p' , and $Q^2 = -(p' - p)^2$. The diagonal matrix elements of the axial current are called the axial charges,

$${}_N \langle p | \bar{q} \gamma_\mu \gamma_5 q | p \rangle_N = 2M s_\mu \Delta q$$

where M and s_μ are respectively the mass and spin vector of the nucleon. The quantities Δq are called “axial charges” because they are the value of the axial form factors at $Q^2 = 0$; that is to say, for example, $G_A^s(Q^2 = 0) = \Delta s$. The connection between the two sets of observables lies in a well-known QCD sum rule for the axial current, namely that the value of the axial form factor at $Q^2 = 0$ is equal to the integral over the polarized parton distribution function measured at $Q^2 = \infty$. For example,

$$\Delta s = G_A^s(Q^2 = 0) = \int_0^1 \Delta s(x, Q^2 = \infty) dx.$$

In this way, the axial charges Δq provide the link between the low-energy elastic scattering measurements of axial form factors and the high-energy deep-inelastic measurements of polarized parton distribution functions.

Of course, there are practical difficulties in the full exploration of this sum rule. No scattering experiment can reach $Q^2 = 0$ or $Q^2 = \infty$, and no deep-inelastic experiment can ever reach $x = 0$. However, the consequences of these difficulties are more severe in some cases than in others. Our inability to reach $Q^2 = \infty$ in the deep-inelastic program means that QCD corrections enter into the sum rule written above, and there is much theoretical experience in calculating these corrections. While the low-energy elastic experiments cannot reach $Q^2 = 0$, there is not expected any divergent behavior of the form factors near $Q^2 = 0$ and so the idea of extrapolating to $Q^2 = 0$ from measurements at low, non-zero Q^2 is not met with alarm. On the other hand, the limitations imposed by our inability to reach $x = 0$ in the deep-inelastic experiments are more problematic. The unpolarized parton distribution functions $q(x)$ are all known to increase rapidly as $x \rightarrow 0$ and there is no calculation of the expected behavior near $x = 0$ to rely upon for an extrapolation from measurements made at $x \neq 0$. Similar comments apply to the polarized parton distributions $\Delta q(x)$. Unpolarized measurements of the parton distributions at HERA have reached very low values of x , nearing $x = 3 \times 10^{-5}$, but the corresponding measurements of the polarized distribution functions, from data at SLAC, CERN, and DESY, only reach $x = 3 \times 10^{-3}$. Therefore, measurements of the axial charges place important constraints on the behavior of the distributions $\Delta q(x)$ in the unmeasurable low- x region.

2 Δs from inclusive leptonic deep-inelastic scattering

As mentioned earlier, the double-spin asymmetries in polarized inclusive leptonic deep-inelastic scattering measure the spin-dependent nucleon structure function g_1 :

$$g_1(x) = \frac{1}{2} \sum_q e_q^2 \Delta q(x).$$

In leading order QCD, these functions take on a scale dependence:

$$g_1(x, Q^2) = \frac{1}{2} \sum_q e_q^2 \Delta q(x, Q^2).$$

In next-to-leading order (NLO) QCD, there are significant radiative corrections and the relation between g_1 and the Δq becomes more complex. In the discussion here, we will limit our attention to the leading-order QCD analysis because the NLO version of the analysis does not change the result (nor the uncertainty) for Δs very much, and the problems to be pointed out exist at all orders, because they are problems coming from the data itself.

We will use the analysis from the SMC Collaboration [2] as a model. They measured $g_1(x, Q^2)$ over a wide

kinematic range, $0.003 < x < 0.70$ and $1.3 < Q^2 < 58.0$. This coverage is not a rectangle, *i.e.* there is a correlation between x and Q^2 in the acceptance of the experiment, and so for a reasonable analysis it is necessary to use QCD to evolve all the data to a single value of Q^2 , in this case $Q^2 = 10 \text{ GeV}^2$. In the process of performing this evolution, a fit function for g_1 is produced. Then, to integrate the distribution g_1 over $0 < x < 1$, it is necessary to extrapolate to $x = 1$ and $x = 0$. The extrapolation to $x = 1$ makes use of the fact that g_1 , being a difference of two quark distributions, must go to 0 as $x \rightarrow 0$. This requirement is satisfied in this analysis by assuming the measured experimental asymmetry to be constant for $x > 0.7$. The extrapolation to $x = 0$, on the other hand, is not straightforward, as the expected behavior of $g_1(x)$ for $x \rightarrow 0$ is unknown. In this analysis, two methods were used. In one, the QCD evolution fit was simply extrapolated to $x = 0$. In another, called the ‘‘Regge extrapolation,’’ the value of g_1 was assumed to be constant for $x < 0.003$. The two values of the integral of g_1 from these two extrapolations are

$$\begin{aligned} \Gamma_1 &= \int_0^1 g_1(x) dx = 0.142 \pm 0.017 \quad \text{‘‘Regge’’} \\ &= 0.130 \pm 0.017 \quad \text{QCD fit.} \end{aligned}$$

This integral is related to the axial charges:

$$\begin{aligned} \Gamma_1 &= \int_0^1 g_1(x) dx = \frac{1}{2} \sum_q e_q^2 \int_0^1 \Delta q(x) dx \\ &= \frac{1}{2} \left[\frac{4}{9} \Delta u + \frac{1}{9} \Delta d + \frac{1}{9} \Delta s \right]. \end{aligned}$$

Now, assuming that $SU(3)_f$ is a valid symmetry of the baryon octet, and using hyperon β decay data, then two other relations between the three axial charges are determined:

$$\Delta u - \Delta d = g_A = F + D \quad \text{and} \quad \Delta u + \Delta d - 2\Delta s = 3F - D$$

where $g_A = 1.2601 \pm 0.0025$ and $F/D = 0.575 \pm 0.016$ (in 1997). Now one may solve for the axial charges, and the results are shown in Table 1. It is well to note that, of course,

Table 1. Results for the axial charges from the SMC analysis [2] of their inclusive DIS data

	‘‘Regge’’	QCD fit
Δu	0.84 ± 0.06	0.80 ± 0.06
Δd	-0.42 ± 0.06	-0.46 ± 0.06
Δs	-0.08 ± 0.06	-0.12 ± 0.06

the error bars quoted here do not include any estimate of the theoretical uncertainty underlying the assumption of $SU(3)_f$ symmetry. They do include an estimate of the uncertainty due to the extrapolations, but of course that is only an estimate because the actual behavior of g_1 is

unknown in the $x \rightarrow 0$ region. The only conclusion to be drawn for Δs from this analysis is that it may be negative, with a value anywhere in the range 0 to -0.2 .

3 $\Delta s(x)$ from semi-inclusive leptonic deep-inelastic scattering

In semi-inclusive deep-inelastic scattering experiments, a leading hadron is observed in coincidence with the scattered lepton, allowing a statistical identification of the struck quark, and thus a measurement of the x -dependence of the individual $\Delta q(x)$ distributions. (Inclusive scattering only measures the total structure function $g_1(x)$.) The HERMES Experiment [3] at DESY was especially designed to make this measurement. HERMES measured double-spin asymmetries in the production of charged hadrons in polarized deep-inelastic scattering of positrons from polarized targets; specifically, the asymmetry in the production of charged pions on targets of hydrogen and deuterium, and of charged kaons in scattering from deuterium. There is no assumption of $SU(3)_f$ symmetry in their analysis. They extract the following quark polarization distributions, over the range $0.023 < x < 0.30$ [4]:

$$\frac{\Delta u}{u}(x) \quad \frac{\Delta d}{d}(x) \quad \frac{\Delta \bar{u}}{\bar{u}}(x) \quad \frac{\Delta \bar{d}}{\bar{d}}(x) \quad \frac{\Delta s}{s}(x)$$

where $\frac{\Delta s}{s}(x)$ is defined to be the sum of $\frac{\Delta s}{s}(x)$ and $\frac{\Delta \bar{s}}{\bar{s}}(x)$.

Within the measured uncertainties, and within the measured x -region, the valence quarks (u and d) are polarized and the sea quarks (\bar{u} , \bar{d} , and s) are unpolarized. The integral value of the measured polarized strange quark distribution is

$${}^{\prime}\Delta s^{\prime} = \int_{x=0.023}^{0.30} \Delta s(x) dx = +0.03 \pm 0.03(\text{stat}) \pm 0.01(\text{syst}).$$

[Note this would only be the true Δs if the integral was over the range $0 < x < 1$.]

Given the fact that the inclusive analysis described in the previous section produced a negative value of Δs , it is natural to ask ‘‘where did the negative Δs go?’’ If the analyses shown of the inclusive and semi-inclusive data are both correct, then all the negative contribution to the value of Δs must come from the unmeasured x -region, that is from $x < 0.023$. That would imply an average value of $\Delta s(x)$ of approximately -5 in the range $x < 0.023$, which is not impossible, as $s(x)$ is of order 20-300 in the range $0.01 < x < 0.001$ [5]. Some very interesting physics indeed would be revealed, if the ‘‘turn on’’ of the strange quark polarization in the low- x region was this dramatic.

Of course, there are other explanations. The invocation of $SU(3)_f$ symmetry in the analysis of the inclusive data is known to be problematic. And the extrapolations to $x = 0$ in those analyses do not have firm theoretical support. It is clear that a *direct* measurement of Δs would serve to clarify these issues.

4 Parity-violating eN elastic scattering

One of the highlights of the current low- and medium-energy electron scattering program is the measurement of the strange vector form factors of the nucleon via parity-violating eN scattering. These measurements are sensitive as well to the non-strange part of the axial form factor, but rather insensitive to the strange axial form factor due to the relative sizes of kinematic factors multiplying the various form factors that contribute to the asymmetry. To be specific, the parity-violating asymmetry observed in these experiments, when the target is a proton, can be expressed as [6]

$$A_p = \left[\frac{-G_F Q^2}{4\pi\alpha\sqrt{2}} \right] \times \frac{\epsilon G_E^\gamma G_E^Z + \tau G_M^\gamma G_M^Z - (1 - 4\sin^2\theta_W)\epsilon' G_M^\gamma G_A^e}{\epsilon (G_E^\gamma)^2 + \tau (G_M^\gamma)^2}$$

where $G_{E(M)}^\gamma$ are the traditional electric (magnetic) form factors of the proton and $G_{E(M)}^Z$ are their weak analogs, $\tau = Q^2/4M_p^2$, M_p is the mass of the proton, $\epsilon = [1 + 2(1 + \tau)\tan^2(\theta/2)]^{-1}$, θ is the electron scattering angle, and $\epsilon' = \sqrt{\tau(1 + \tau)(1 - \epsilon^2)}$. Lastly, G_A^e is the effective axial form factor seen in electron scattering:

$$G_A^e = -G_A^{CC}(1 + R_A^{T=1}) + G_A^s + R_A^{T=0}.$$

Here, G_A^{CC} is the non-strange (CC) axial form factor, G_A^s is the strange axial form factor, and the terms $R_A^{T=0,1}$ represent electroweak radiative corrections [6, 7, 8, 9]. The presence of these radiative corrections clouds the interpretation of the axial term extracted from these experiments. To solve this problem, the SAMPLE [10] experiment measured also the same asymmetry on a deuterium target, in which case the relative kinematic factors of the non-strange ($T = 1$) and strange ($T = 0$) parts of the axial form factor are changed, allowing a separation of the two. However, one may show that this does not help in identifying the value of G_A^s , because the relative size of the kinematic factors for G_M^s and G_A^s remain the same for either target:

$$\frac{\partial G_M^s}{\partial G_A^s} = -(1 - 4\sin^2\theta_W) \frac{\epsilon'}{\tau} \approx -\frac{1}{2} \text{ for SAMPLE.}$$

Therefore, parity-violating eN scattering experiments can only establish a relationship between the strange magnetic and axial form factors, they cannot measure them separately.

5 Combining νN and parity-violating eN elastic data

Recently it has been demonstrated [11] that the best (perhaps only) way to obtain all three strange quark form factors (electric, magnetic and axial) is through a combination of low energy νN and parity violating eN elastic

Table 2. Two solutions for the strange form factors at $Q^2 = 0.5 \text{ GeV}^2$ produced from the E734 and HAPPEX data. (Table from [11])

	Solution 1	Solution 2
G_E^s	0.02 ± 0.09	0.37 ± 0.04
G_M^s	0.00 ± 0.21	-0.87 ± 0.11
G_A^s	-0.09 ± 0.05	0.28 ± 0.10

scattering. This prescription is briefly summarized here. Using the known values for the electric, magnetic and non-strange (CC) axial form factors of the proton and neutron, one may take the difference of the νp and $\bar{\nu} p$ elastic cross sections and show it to be a function only of the strange magnetic and axial form factors, G_M^s and G_A^s . At the same time, the sum of the νp and $\bar{\nu} p$ elastic cross sections can be shown to be a function only of the strange electric and magnetic form factors, G_E^s and G_M^s . Measurements of forward-angle parity-violating ep elastic scattering are largely functions only of G_E^s and G_M^s as well. Therefore, combining these three kinds of data can determine all three strange form factors. At the present time, there is only sufficient data at $Q^2 = 0.5 \text{ GeV}^2$ to make such a determination. In [11] it is shown that by combining the E734 [12] results with the HAPPEX [13] data from JLab, there are two possible solutions at $Q^2 = 0.5 \text{ GeV}^2$, as summarized in Table 2. In the long run, additional experimentation (already on the schedule at JLab) will select one of these solution sets, but there are already several good reasons to favor Solution 1 over Solution 2. The value of the strange electric form factor G_E^s must approach zero as $Q^2 \rightarrow 0$, and Solution 1 is consistent with that requirement. The value of G_A^s in Solution 1 is consistent with the estimated value of $G_A^s(Q^2 = 0) = \Delta s \approx -0.1$ from deep-inelastic data, whereas that found in Solution 2 is much larger and of a different sign. The value of G_M^s in Solution 1 is consistent with that measured by SAMPLE [10] at $Q^2 = 0.1 \text{ (GeV/c)}^2$, whereas the value in Solution 2 is much larger in magnitude. Finally, the value of G_M^s in Solution 1 is consistent with the value of $G_M^s(Q^2 = 0) = -0.051 \pm 0.021 \mu_N$ predicted recently from lattice QCD [14]. It seems nearly certain that Solution 1 is the true physical solution. Future experimentation will in any event select the correct solution set.

Additional data from the G^0 Experiment [15], recently collected and in the process of analysis, will allow the extraction of the three strange form factors when combined with the E734 data. However, it is unlikely that knowledge of the strange axial form factor over the range $0.5 < Q^2 < 1.0 \text{ GeV}^2$ will prove sufficient for the extrapolation to $Q^2 = 0$ needed for a determination of Δs . New neutrino data are needed at lower Q^2 to permit a good determination of Δs .

6 A future experiment to measure the strange axial charge

Even if the program I have described determines the strange axial form factor down to $Q^2 = 0.45 \text{ GeV}^2$ suc-

cessfully, it almost certainly will not determine the Q^2 -dependence sufficiently for an extrapolation down to $Q^2 = 0$. Also, questions remain about the normalization of the E734 data. Most of their target protons were inside of carbon nuclei, and there was not much known about nuclear transparency in the mid-1980's. The E734 collaboration did make a correction for transparency effects, but this issue needs to be revisited if we continue to use the E734 data.

A new experiment [16] has been proposed to measure elastic and quasi-elastic neutrino-nucleon scattering to sufficiently low Q^2 to determine the strange axial charge, Δs . The FINeSSE Collaboration proposes to measure the NC to CC neutrino scattering ratio

$$R_{NC/CC} = \frac{\sigma(\nu p \rightarrow \nu p)}{\sigma(\nu n \rightarrow \mu^- p)}$$

and from it extract the strange axial form factor down to $Q^2 = 0.2 \text{ GeV}^2$. The numerator in this ratio is sensitive to the full axial form factor, $-G^{CC} + G_A^s$, while the denominator is sensitive to only G_A^{CC} . The processes in the numerator and denominator have unique charged particle final state signatures. This ratio is largely insensitive to uncertainties in neutrino flux, detector efficiency and nuclear target effects. A 6% measurement of $R_{NC/CC}$ down to $Q^2 = 0.2 \text{ GeV}^2$ provides a ± 0.04 measurement of Δs .

Acknowledgements. This work was supported by the US Department of Energy.

References

1. L.A. Ahrens et al.: Phys. Rev. D **34**, 75 (1986)
2. B. Adeva et al.: Phys. Lett. B **412**, 414 (1997)
3. K. Akerstaff et al.: Nucl. Instrum. Meth. A **417**, 230 (1998)
4. A. Airapetian et al.: Phys. Rev. Lett. **92**, 012005 (2004)
5. J. Pumplin et al.: JHEP **0207**, 012 (2002)
6. M.J. Musolf et al.: Phys. Rep. **239**, 1 (1994)
7. M.J. Musolf, B.R. Holstein: Phys. Lett. B **242**, 461 (1990)
8. M.J. Musolf, T.W. Donnelly: Nucl. Phys. A **546**, 509 (1992)
9. S.-L. Zhu et al.: Phys. Rev. D **66**, 034021 (2002)
10. D.T. Spayde et al.: Phys. Lett. B **583**, 79 (2004); T.M. Ito et al.: Phys. Rev. Lett. **92**, 102003 (2004)
11. S.F. Pate: Phys. Rev. Lett. **92**, 082002 (2004)
12. L.A. Ahrens et al.: Phys. Rev. D **35**, 785 (1987)
13. K.A. Aniol et al.: Phys. Lett. B **509**, 211 (2001)
14. D.B. Leinweber et al.: submitted to Phys. Rev. Lett. [arXiv:hep-lat/0406002]
15. G^0 Experiment, <http://www.npl.uiuc.edu/exp/G0/>, Doug Beck, spokesperson
16. FINeSSE Proposal, B. Fleming, R. Tayloe et al.: [arXiv:hep-ex/0402007]